Exercise 15

Differentiate.

$$f(\theta) = \theta \cos\theta \sin\theta$$

Solution

Use the product rule to differentiate $f(\theta)$.

$$\begin{aligned}
f'(\theta) &= \frac{d}{d\theta}[f(\theta)] \\
&= \frac{d}{d\theta}(\theta \cos\theta \sin\theta) \\
&= \left[\frac{d}{d\theta}(\theta)\right] \cos\theta \sin\theta + \theta \left[\frac{d}{d\theta}(\cos\theta)\right] \sin\theta + \theta \cos\theta \left[\frac{d}{d\theta}(\sin\theta)\right] \\
&= (1)\cos\theta \sin\theta + \theta(-\sin\theta)\sin\theta + \theta\cos\theta(\cos\theta) \\
&= \cos\theta\sin\theta - \theta\sin^2\theta + \theta\cos^2\theta \\
&= \frac{1}{2}(2\sin\theta\cos\theta) + \theta(\cos^2\theta - \sin^2\theta) \\
&= \frac{1}{2}\sin 2\theta + \theta\cos 2\theta
\end{aligned}$$